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DYNAMIC ASPECTS OF DAMPED ELASTIC IMPACT
PART 2: INFLUENCE OF INITIAL INDENTATION VELOCITY

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Abstract: The paper presents kinematical and dynamical aspects concerning damped elastic collision between two spheres. The characteristic parameters of system dynamic behaviour are the coefficient of restitution and the initial indentation velocity. The presented results are obtained by integrating a non-linear ordinary differential equation, made in two stages: the first step was an analytical integration followed by a numerical one. The work is parted in two: the first part presents the influence of the coefficient of restitution and the second part is dedicated to the influence of initial indentation velocity upon kinematical and dynamical system parameters.

1. INTRODUCTION.

The models used in the study of collision were presented in Part I of the paper that also mentioned the main directions for the study of the problem, [1÷8]. It has been shown that one of the most widely used models is the one proposed by Lankarani and Nikravesh, [4]. Following Hunt and Crossley model, [3], Lankarani and Nikravesh found the equation describing the dynamic behaviour during non-damped collision of two elastic spheres. The phenomenon is expressed by the following equation:

$$F = K\delta^n\left[1+\frac{3(1-e^2)}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{1}$$

Where $\dot{\delta}^{(-)}$ represents the initial relative velocity, e is the coefficient of restitution, defined as the ratio, taken with changed sign, between the relative velocities at the end of collision and at the beginning of impact. The exponent $n=3/2$ is characteristic to the displacement-force dependence in the case of Hertzian point contact and K is a coefficient considering the elastic properties and the local geometry of the contacting regions. As mentioned in Part I of the paper, the authors found an original method for integration of equation (1). As it can be seen, for bodies with given geometry and elastic properties, the parameters influencing the dynamic behaviour of the system are the coefficient of restitution e and the initial impact velocity $\dot{\delta}^{(-)}$.

$$\dot{\delta}^{(-)} = V_i^{(-)} - V_j^{(-)} \tag{2}$$

In Part I it was proven the influence manifested by the coefficient of restitution upon different kinematical and dynamical parameters. Part II of the work presents the influence of initial indentation velocity upon these parameters.

It must be revealed that following the integration of equation (1), finding the velocity after impact and computing the coefficient of restitution, one can observe that a different value for the coefficient of restitution is found, compared to the initial considered one. This fact narrows the domain where the equation (1) may be applied to the quasielastic impact cases, that is for $e>0.8$. However, this restriction is not a major impediment since in engineering applications there are numerous situations when the collision can be considered as quasielastic. Recently, Flores et al., [5] showed that a convenient adjustment of the coefficient of the term from parenthesis eliminates this inconvenience.

2. THE EFFECT OF INITIAL INDENTATION VELOCITY

For two different values of the coefficient of restitution, $e = 0.7$ and $e = 0.9$, there were considered the cases when the initial indentation velocity has the values

$v_0 = \dot{\delta}^{(-)} = \{1.0; 2.0; 3.0; 4.0; 5.0\}(m/sec)$.

There were considered two values for the coefficient of restitution in order to show up which of the next two parameters, namely:

- The coefficient of restitution
- The initial indentation velocity

demonstrates a stronger influence upon kinematical and dynamical parameters of the system.

There were graphically represented the following variations:

- In Fig. 1, the hysteresis curves ;
- In Fig. 2, the contact force variation versus time;
- In Fig. 3, the normal approach variation as a function of time;
- In Fig. 4, the relative velocity variation;
- In Fig. 5, the relative acceleration variation;
- In Fig. 6, the Poincaré’s map;
- In Fig. 7, the variation of compression time and contact time versus initial velocity v_0
- In Fig. 8, the variation of the ratio between the detaching time and contact time versus initial velocity, v_0;
- In Fig. 9, the variation of the ratio between the detaching time and compression time versus initial velocity, v_0;
- In Fig. 10, the variation of maximum compression force considering damping, versus initial velocity v_0;
- In Fig. 11, the variation of the ratio between maximum contact force for damped collision and maximum contact force without damping, (Hertzian contact) for different values of the coefficient of restitution.

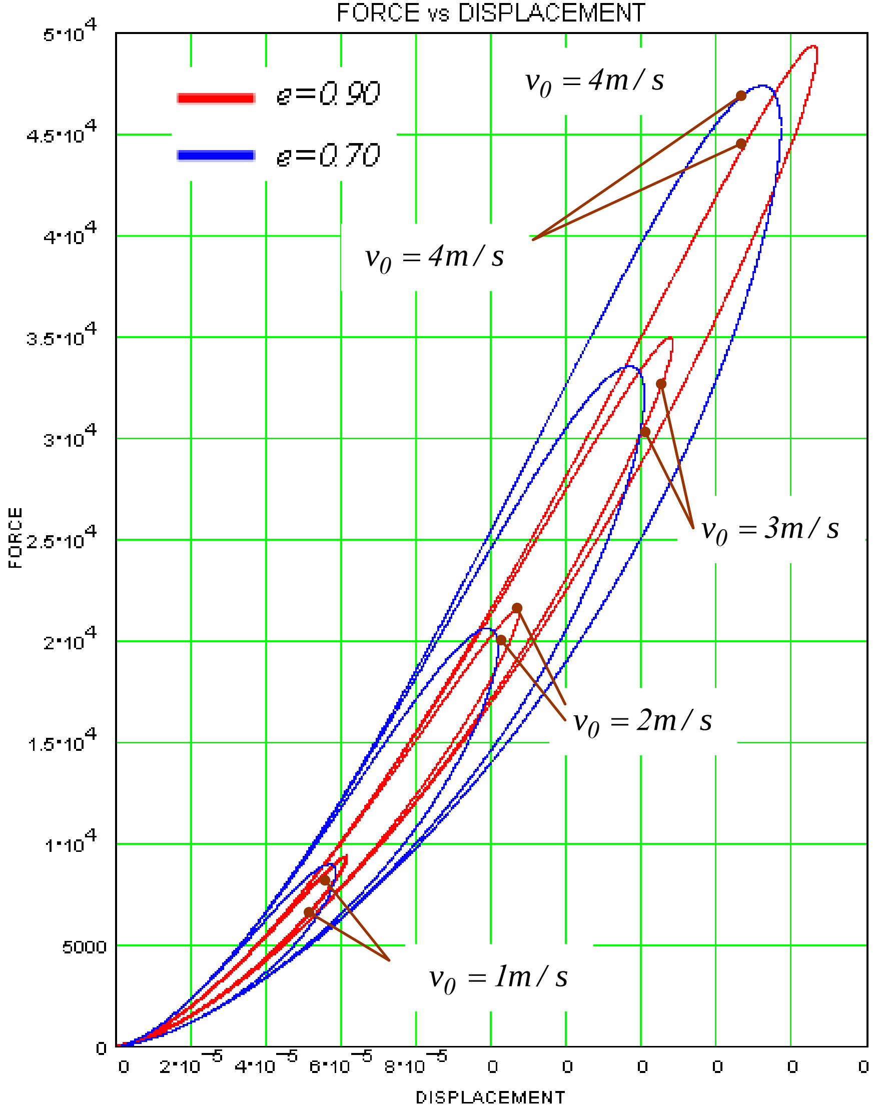

Figure 1. Hysteresis curves

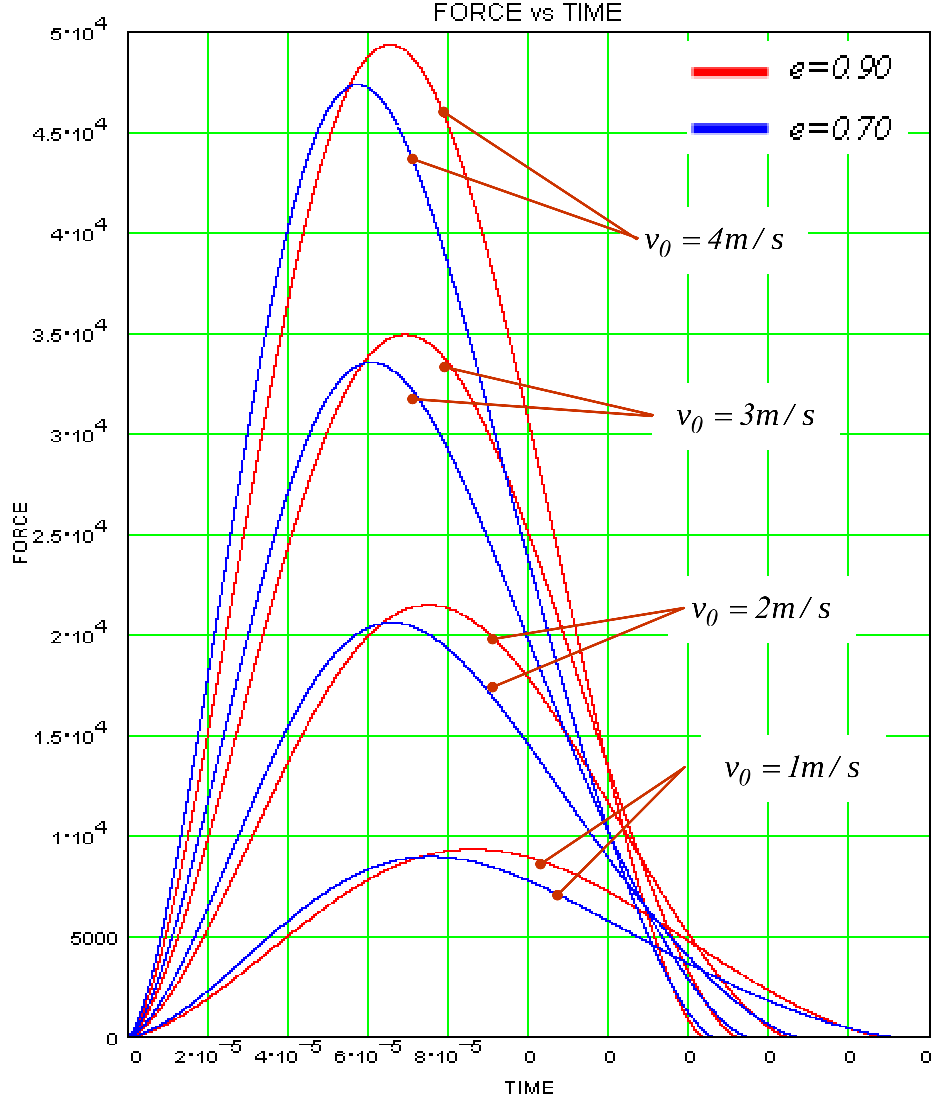

Figure 2. Contact force variation versus time

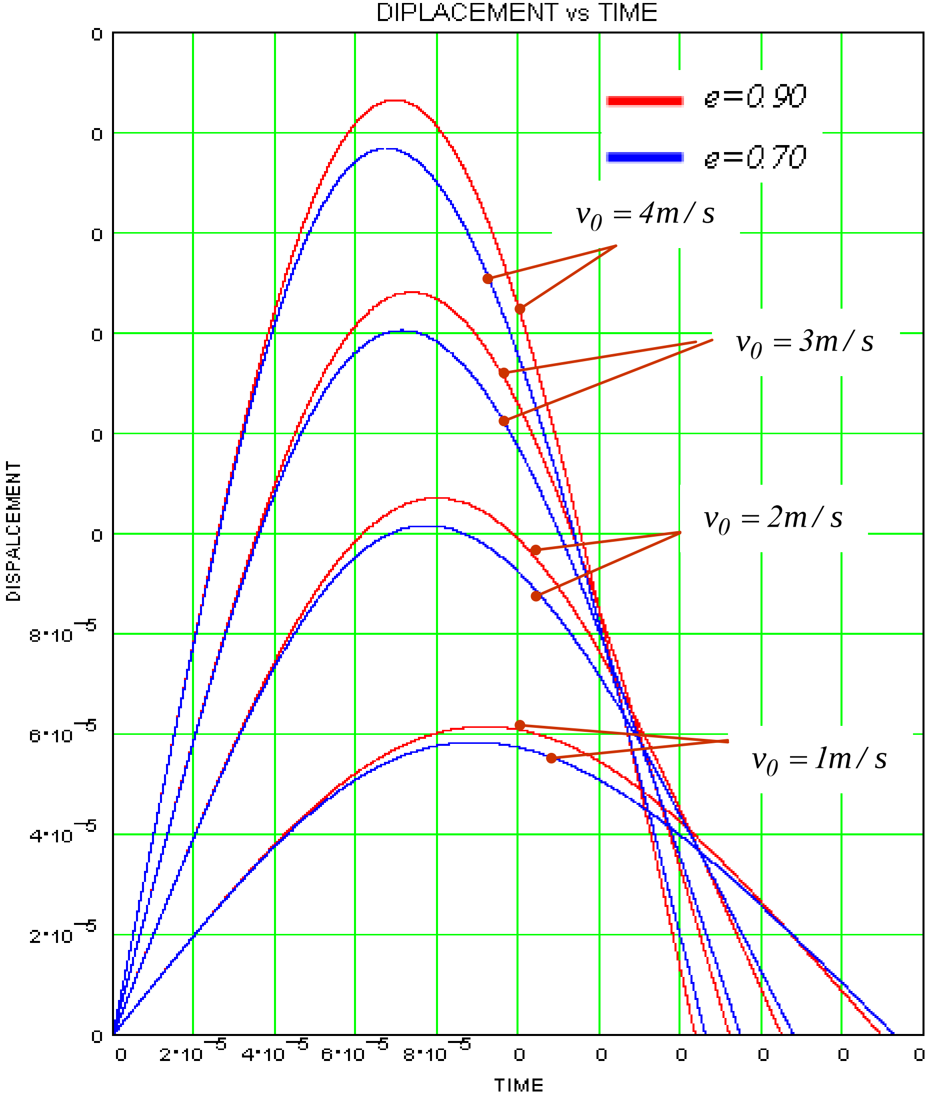

Figure 3. Normal approach variation versus time

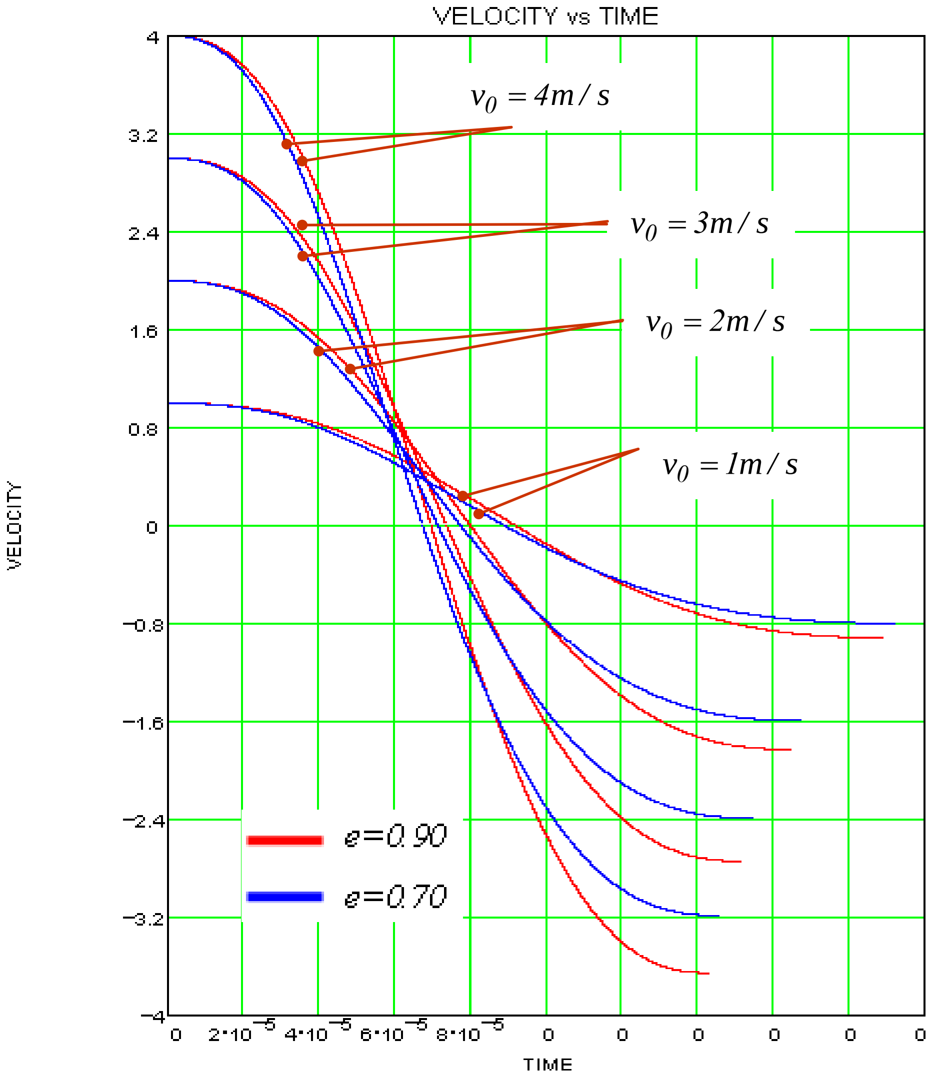

Figure 4. Relative velocity variation versus time

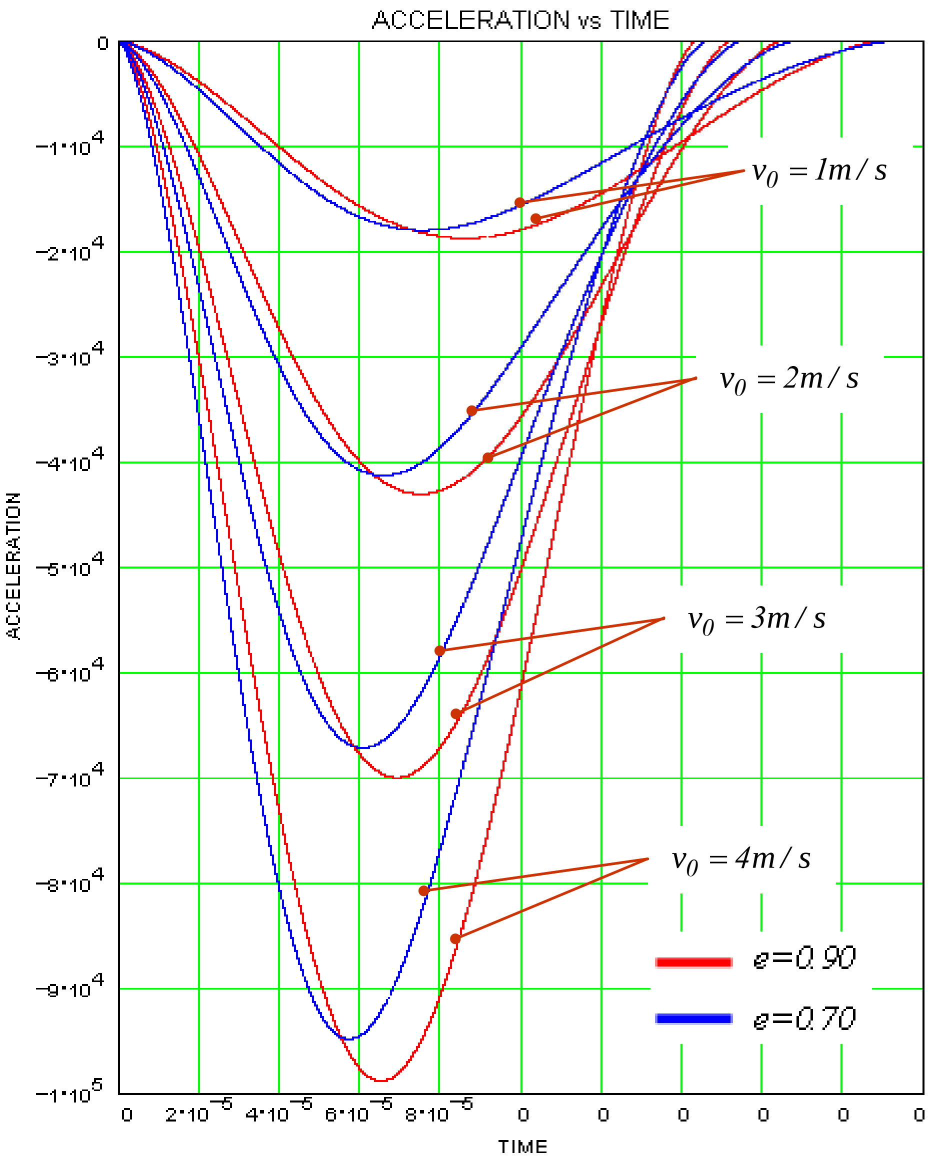

Figure 5. Relative acceleration variation versus time

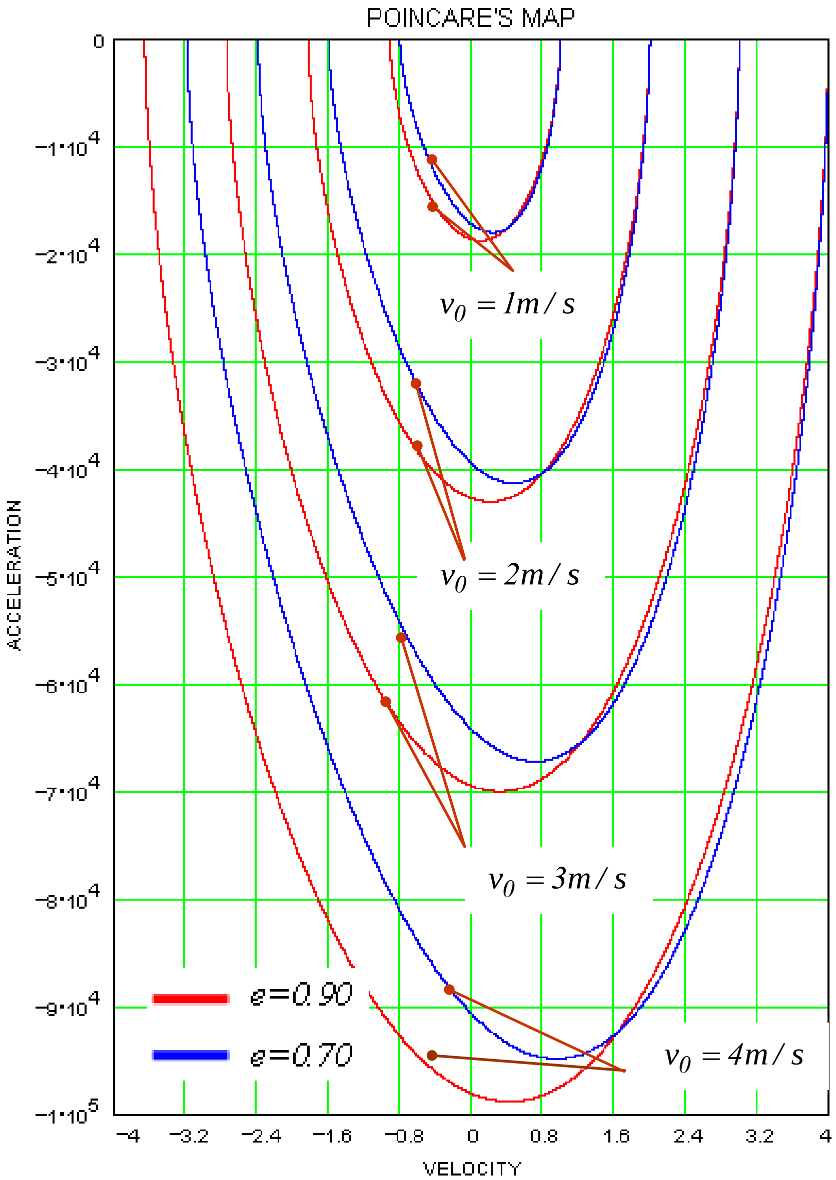

Figure 6. Poincaré's map

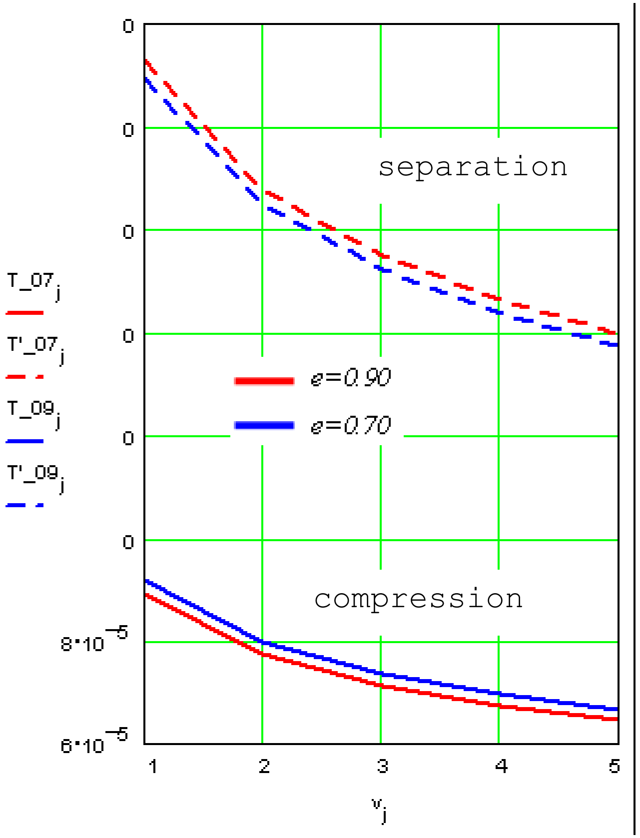

Figure 7. Variation of compression time and contact time versus initial velocity v_0

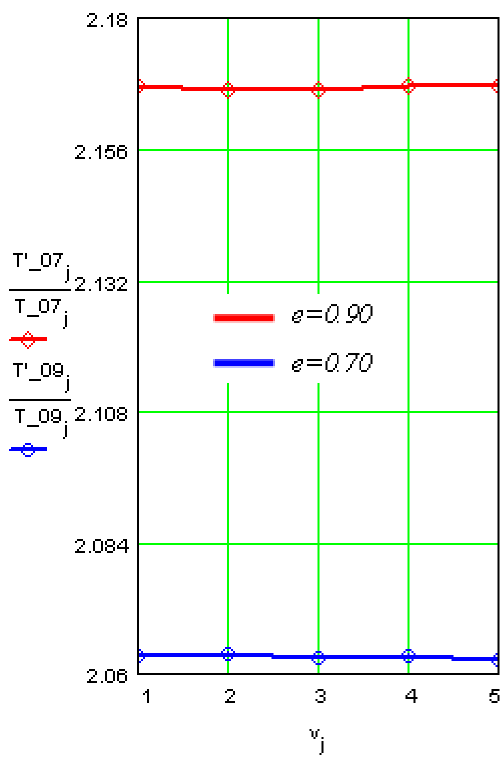

Figure 8. Variation of the ratio between the detaching time and contact time versus initial velocity, v_0

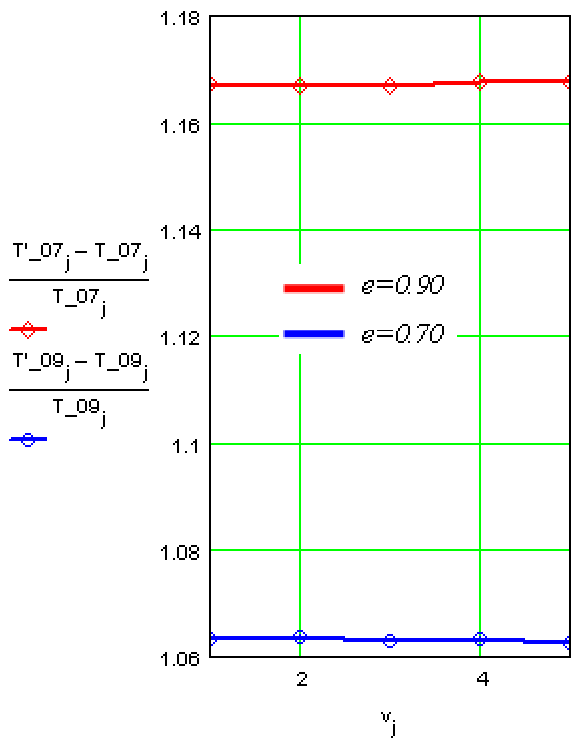

Figure 9. Variation of the ratio between the detaching time and compression time versus initial velocity, v_0

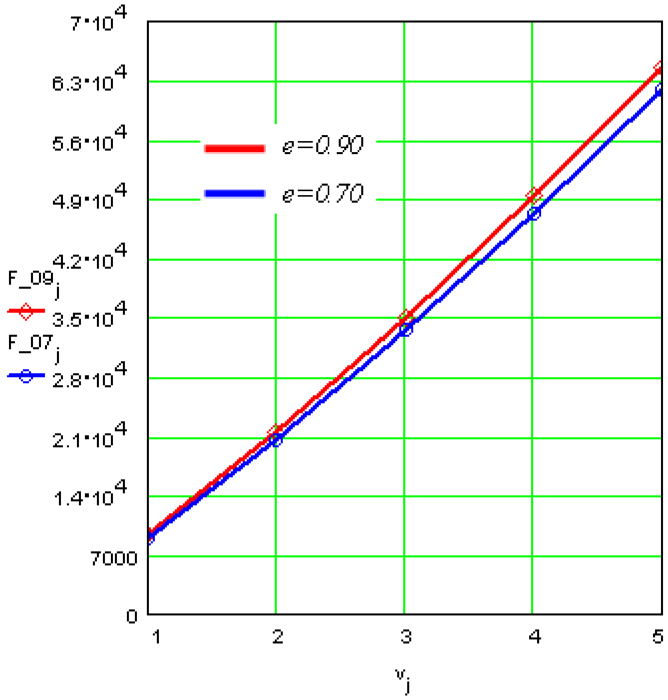

Figure 10. Variation of maximum compression force with damping, versus initial velocity v_0

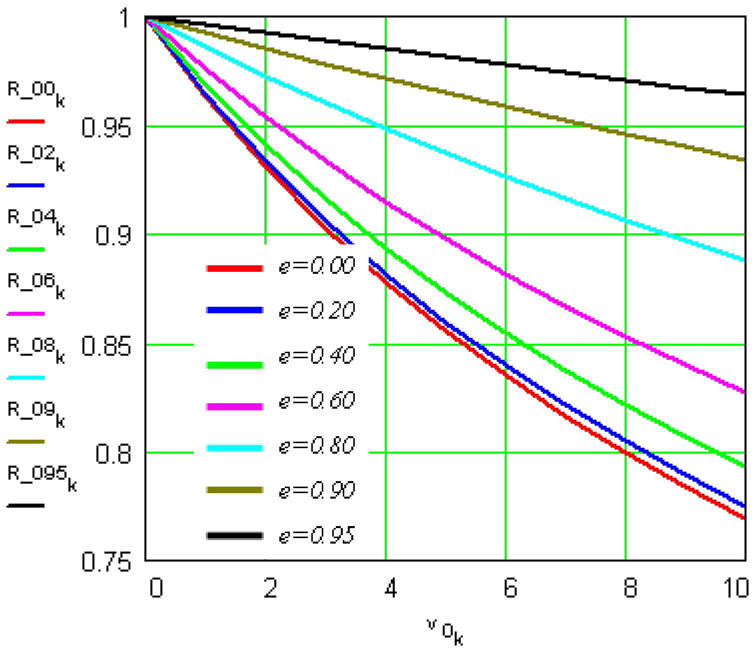

Figure 11. Variation of the ratio between maximum contact force for damped collision and maximum contact force without damping, (Hertzian contact) for different values of the coefficient of restitution

3. CONCLUSIONS

The increase of the initial indentation velocity carries out the increase of the area of hysteresis loop and therefore, the increase of the dissipative work.

Together to the increase of initial velocity, it also increase the following parameters: maximum impact force, maximum approach, separation velocity and maximum acceleration.

With reference to the initial impact velocity upon the duration of compression and separation phases, the increase of the velocity leads to reduced times for both phases. One can notice that the influence is stronger upon separation time.

The coefficient of restitution, COR, doesn't basically influence the compression and separation times.

The ratio between the compression time and the separation time is practically independent of the initial indentation velocity, but presents greater values for greater COR values.

The initial indentation velocity affects strongly the maximum impact force. It can be stated that for low impact velocity values, a quasi-linear dependence occurs between the maximum impact force and the initial impact velocity.

The ratio between the maximum impact force for damped collision and the maximum un-damped impact force decreases with increasing impact velocity. When COR is in close proximity to *1*, this aspect is more obvious.

One must notice that the above conclusions are drawn analyzing the Lankarani-Nikravesh model but for a reasonable employment, it must be considered that the model is valid for values of COR greater than *0.8.*

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